Field Oscillations in a Micromaser with Injected Atomic Coherence

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Abstract

The electric field in a lossless, regularly-pumped micromaser with injected atomic coherence can undergo a period 2 oscillation in the steady state. The field changes its value after a single atom passes through the micromaser cavity, but returns to its original value after a second atom travels through. We give a simple explanation for this phenomenon in terms of tangent and cotangent states. We also examine the effect of cavity damping on this steady state.

1 Introduction

A micromaser consists of a high-Q microwave cavity and a stream of excited Rydberg atoms which pumps the field in the cavity. At any given time, the cavity is either empty or contains one atom. Because the field is pumped by only a single atom at a time, the micromaser is a highly quantum mechanical system. Experiments conducted at the Max Planck Institute for Quantum Optics in Garching have seen such quantum phenomena as sub-Poissonian photon statistics and quantum collapses and revivals [1, 2].

Extensive theoretical work on the micromaser has shown the existence of numerous interesting features. Among them are trapping states, which separate the dynamics into noninteracting blocks of photon numbers [3], and the dependence of steady state of the field on the injection statistics of the atoms [4]. The latter occured as part of more general analysis of pump noise in lasers and masers, and how to minimize it [5, 6]. Despite all of this, however, there are still some surprises. One, found only recently, involves the behavior of the electric field inside the cavity. It has been found by Briegel, Englert, and Scully [7], and independently by Herzog and Bergou [8], that the passage of a single atom can cause the sign of the electric field to flip. They demonstrated this by calculating the two-time field-field correlation function and showing that it can change sign. For a micromaser with regular pumping (equal time intervals between atoms) this effect causes a splitting of the spectrum into several equidistant peaks.

In this work the atoms were injected into the cavity in their upper states. Another kind of field oscillation was found by Slosser and Meystre when the atoms

are injected in a coherent superposition of their upper and lower states [9]. Considering a lossless micromaser, they found steady states for the field which they called tangent and cotangent states (the names refer to the form of coefficients when the states are expanded in the number-state basis), which are confined between trapping states [3]. They then went on to examine the dynamics of the system when the photon numbers were limited by two trapping states. They found,numerically, that if other trapping states intervened between the original two, there were states which one might call steady states of period 2; these states are mapped back onto themselves not by the interaction with one atom, as a normal steady state would be, but by the interaction with two atoms. The electric field of such a state oscillates between two values; it changes its value after the passage of one atom and returns to its original value after the passage of a second.

Here we would like to give a simple explanation for the oscillations seen by Slosser and Meystre. This explanation hinges on a slight difference in the behavior of tangent and cotangent states after the passage of an atom. In addition we wish to see how the oscillatory behavior is affected by the presence of losses. In the next section we examine the lossless case and in the following section losses are included.

2 Lossless Micromaser

We shall consider a micromaser in which the pumping atoms are injected at regular time intervals, with the time between atoms being T. Strictly speaking, this assumption is not necessary for our analysis in the lossless case to be valid. However, when we do consider losses it will be necessary to make a particular choice for the injection statistics, and this is the choice we shall make. Each atom interacts with the field for a time, τ, which is much smaller than T. Between atoms the field evolves freely, while during the time an atom is in the cavity the interaction of the field and the atom is governed by the Jaynes-Cummings Hamiltonian

$$H = \omega a^{\dagger} a + \frac{1}{2}\omega(\sigma_3 + I) + g(a^{\dagger}\sigma^{-} + a\sigma^{+}), \tag{1}$$

where ω is both the frequency of the cavity mode and the transition frequency of the atom, and g is the atom-field coupling constant. The two-level atom is described by a two-dimensional state space on which the Pauli matrixes in the Hamiltonian act. We denote the upper state of the atom by $|a\rangle$ and the lower by $|b\rangle$. Note that we have assumed that the atom and the cavity mode are in resonance.

Let us now suppose that the atom is initially in the state $\alpha|a\rangle + \beta|b\rangle$, and the field is in the state

$$|f\rangle = \sum_{n=0}^{\infty} d_n |n\rangle, \tag{2}$$

where $|n\rangle$ is a photon number state with n photons. If this state evolves for a time τ under the action of the Hamiltonian in Eq. (1), then the resulting state

in the interaction picture is

$$|f\rangle(\alpha|a\rangle+\beta|b\rangle) \quad\rightarrow\quad \sum_{n=0}^{\infty} d_n(\alpha c_{n+1}|n\rangle - i\beta s_n|n-1\rangle)|a\rangle + \sum_{n=0}^{\infty} d_n(\beta c_n|n\rangle - i\alpha s_{n+1}|n+1\rangle)|b\rangle, \tag{3}$$

where

$$s_n = \sin(g\tau\sqrt{n}) \qquad\qquad c_n = \cos(g\tau\sqrt{n}). \tag{4}$$

Slosser and Meystre found the tangent and cotangent states by demanding that the state on the right-hand side of this equation be a product of the original field state and an atomic state, i. e. that

$$|f\rangle(\alpha|a\rangle+\beta|b\rangle) \rightarrow |f\rangle(\alpha'|a\rangle+\beta'|b\rangle). \tag{5}$$

This condition guarantees that the field state is unchanged by the passage of an atom. They found that this condition is satisfied if either $\alpha' = -\alpha$, $\beta' = \beta$, and

$$d_n = -i\frac{\alpha}{\beta}\cot(g\tau\sqrt{n}/2)d_{n-1}, \tag{6}$$

(cotangent state) or if $\alpha' = \alpha$, $\beta' = -\beta$, and

$$d_n = i\frac{\alpha}{\beta}\tan(g\tau\sqrt{n}/2)d_{n-1}, \tag{7}$$

(tangent state). The tangent and cotangent states are normalizeable only if the sums over number states are restricted to a finite range. Expressing both states as

$$|f\rangle = \sum_{n=N_d}^{N_u} d_n|n\rangle, \tag{8}$$

and imposing the conditions $d_{N_d-1}=0$ and $d_{N_u+1}=0$, we find that

$$g\tau\sqrt{N_u+1} = p\pi \qquad\qquad g\tau\sqrt{N_d} = q\pi, \tag{9}$$

where p and q are integers. For tangent states p is even and q is odd, while for cotangent states p is odd and q is even.

The results of the preceding paragraph imply that for a tangent state, $|f_t\rangle$,

$$|f_t\rangle(\alpha|a\rangle+\beta|b\rangle) \rightarrow |f_t\rangle(\alpha|a\rangle-\beta|b\rangle), \tag{10}$$

and for a cotangent state, $|f_c\rangle$,

$$|f_c\rangle(\alpha|a\rangle+\beta|b\rangle) \rightarrow -|f_c\rangle(\alpha|a\rangle-\beta|b\rangle). \tag{11}$$

If we have a state which is a superposition of a tangent and a cotangent state, $\xi_t|f_t\rangle + \xi_c|f_c\rangle$, see Figure 1, then after one atom passes through the cavity it

becomes $\xi_t|f_t\rangle - \xi_c|f_c\rangle$ (where the states are defined up to a common sign), and after a second atom it returns to the original state. This state is then periodic with period 2 (where time is measured in units of T).

How can we see this oscillation of the state? Because of the conditions on N_d and N_u, tangent and cotangent states must exist in nonoverlapping blocks of number states. This means that the oscillations will not show up in observables which commute with the number operator. On the other hand, if the blocks are adjacent, then the electric field operator, which is proportional to $(a^\dagger - a)$, can connect the two blocks, and the effect of the relative sign flip between them will manifest itself as an oscillation in the expectation value of the field. In order to see this explicitly let us look at the situation considered by Slosser and Meystre when $g\tau = \pi$, and there is a cotangent state at $n = 0$ (for certain values of the interaction time the vacuum satisfies the conditions necessary to be a cotangent state), a tangent state between 1 and 3, and a cotangent state between 4 and 8. The expectation value of the electric field in this state is

$$\langle E\rangle = i\sqrt{\frac{\omega}{2V}}\sum_{n=1}^{8}\sqrt{n}(\rho_{n-1,n} - \rho_{n,n-1}), \tag{12}$$

where V is the quantization volume, and ρ is the field density matrix. After passage of an atom a relative sign is introduced between the tangent state and the two cotangent states. This means that the density matrix elements between different blocks, ρ_{01}, ρ_{34}, and their complex conjugates, change sign while the others do not. This causes $\langle E\rangle$ to change. The passage of a second atom causes these density matrix elements to flip sign again, which restores $\langle E\rangle$ to its original value.

The periodicity of the state, then, does have observable effects. Besides the field one could also look at an observables such as $Y_1 = [a^2 + (a^\dagger)^2]/2$, $Y_2 = i[(a^\dagger)^2 - a^2]/2$ which appear in the study of some forms of higher- order squeezing, and which also connect blocks. Because these connect the number states $|n\rangle$ and $|n+2\rangle$, rather than $|n\rangle$ and $|n+1\rangle$ as does the electric field operator, it will lead to a larger number of density matrix elements which flip sign and can thereby produce a larger effect.

3 Micromaser with Losses

We now include losses in our system in order to see whether the steady-state oscillations which we discussed in the previous section will survive under under these more realistic conditions. Because the atom-field interaction time τ is much shorter than the time the cavity is empty,T, we shall ignore the effect of field losses during the times atoms are in the cavity. The decay of the micromaser field for the cavity at zero temperature is described by the master equation

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = -\frac{1}{2}\gamma\left(a^\dagger a\hat{\rho} + \hat{\rho}a^\dagger a - 2a\hat{\rho}a^\dagger\right)$$

which has the solution in the number-state representation

$$\rho_{mn}(t) = \mathrm{e}^{-\frac{\gamma t(m+n)}{2}} \sum_{l} \sqrt{\frac{(m+l)!}{m!}\frac{(n+l)!}{n!}} \frac{(1-\mathrm{e}^{-\gamma t})^{l}}{l!} \rho_{m+l,n+l}(0). \tag{13}$$

We shall numerically simulate the system by using the Jaynes-Cummings dynamics to describe the atom-field interaction and the loss master equation to describe the field during the periods during which the cavity is empty. The values of the micromaser parameters which we used in our numerical simulations were chosen to be as close as possible to those which occur in actual experiments [2]. The atom-field coupling constant g was set to $4.4 \times 10^{4} Hz$, the time between two consecutive atoms was set to $T = 6.666 \times 10^{-3} s$, the cavity loss coefficient γ was set to $5s^{-1}$, which provided cavity photon storage time $T_{cav} = 0.2s$, and number of atoms passing through the cavity during a single decay time was taken to be $N_{ex} = 30$. The atom-field interaction time τ was varied in order to provide needed trapping condition, $g\tau = \pi$. The values of τ used in simulations came very close to the values in actual experiment [2].

Two-level atoms in the coherent superposition $\alpha|a\rangle + \beta|b\rangle$ are injected regularly into the micromaser cavity. The parameters α and β were chosen real with $\alpha = 0.9$. The cavity field was prepared initially in a state with a cotangent state at $n = 0$, a tangent state between 1 and 3, and a cotangent state between 4 and 8, using the same values of α, β as the pumping atoms. The starting values for the recurrence formulas (6,7) were optimized in order to provide large magnitudes of the observables of interest. Then the first atom was injected at $t = 0$, followed by much longer time period T during which the field decayed. Then the second atom was injected followed by the cavity decay period and so on. The expectation values of observables were calculated after a decay time interval just before the injection of the next atom.

The expectation values of the electric field, operator Y_1 and Y_2 for the case are plotted in Figure 2. The interaction time is $\tau = 7.14 \times 10^{-5} s$ giving $\frac{\tau}{T} \cong 10^{-2}$, which justifies the assumption that we can neglect losses during the atom-field interaction. The pumping parameter $\theta_{int} = g\tau\sqrt{N_{ex}} \cong 17.2$. The mean value of each operator exhibits period-two oscillations, but with decreasing magnitude, because of the presence of damping. These expectation values eventually reach steady state values which do not exhibit oscillations. In addition, we found that the field approaches its steady state extremely slowly. This is caused by the presence of the trapping states which separate the total Fock space for the Jaynes-Cummings evolution into independent blocks. The probability flow between the subspaces in which the cotangent and tangent states are located occurs only because of the loss process, and it is very small.

We now look at the density matrix itself. The photon number distribution of the field, given by the diagonal density matrix elements, after the passage of different numbers of atoms is shown in Figure 3. Note that the presence of damping drives the field toward lower photon numbers. In Figure 4 are plotted absolute values of the cavity field density matrix elements. Initially, because the system starts in a pure state, the density matrix has off-diagonal peaks. Because

of the loss process, the off-diagonal elements of the density matrix decay as it approaches its new steady state. This new steady state is very different from the initial cotangent-tangent state, and this explains the deterioration of the oscillations in the expectation values of the operators, because they depend on the presence of the tangent and cotangent states which exhibit a relative phase oscillation.

Our conclusions on the effects of damping are consistent with the results of Slosser, Meystre, and Wright [10]. They considered a micromaser with Poissonian pumping and found that tangent and cotangent states maintain their integrity only in the limit of very large N_{ex}, i. e. very weak damping. For the damping which occurs in experiments, the damping drastically alters the character of the steady-state field.

4 Conclusion

We have shown that the period-two oscillations which exist in a lossless micromaser pumped by atoms in a coherent superposition of states are due to a relative sign change undergone by tangent and cotangent states upon passage of an atom through the cavity. These oscillations can only occur in observables, such as the electric field, which do not commute with the photon number operator. The addition of damping changes the oscillations from a steady-state to a transient phenomenon, but one with a rather long lifetime.

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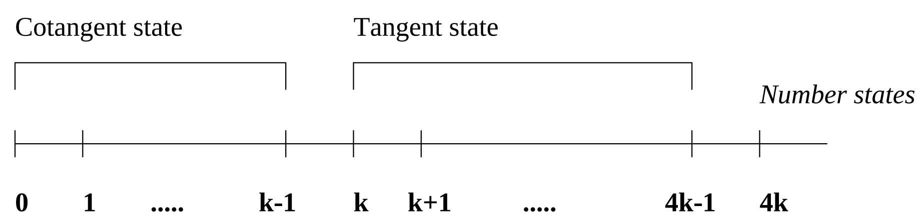
Figure 1: Cotangent and tangent states. The state $|k\rangle$ is π trapping state, $g\tau\sqrt{k} = \pi$.

Figure 2: (a): Evolution of the expectation value of electric field $\langle E\rangle$ with respect to the number of atoms n which passed through the cavity, (b) shows the evolution of $\langle Y_1\rangle$ and (c) shows the evolution of $\langle Y_2\rangle$.

Figure 3: Photon number distribution $P_n = \rho_{nn}$ of the cavity field for the initial state (a), after interaction with 20 atoms (b), and after interaction with 100 atoms (c).

Figure 4: Moduli of cavity field density matrix, (a) shows initial state, the nondiagonal peaks are apparent. (b) shows the state after interaction with 10 atoms, (c) after 20 atoms, (d) after 100 atoms.


Cotangent state
Tangent state
Number states
0
1
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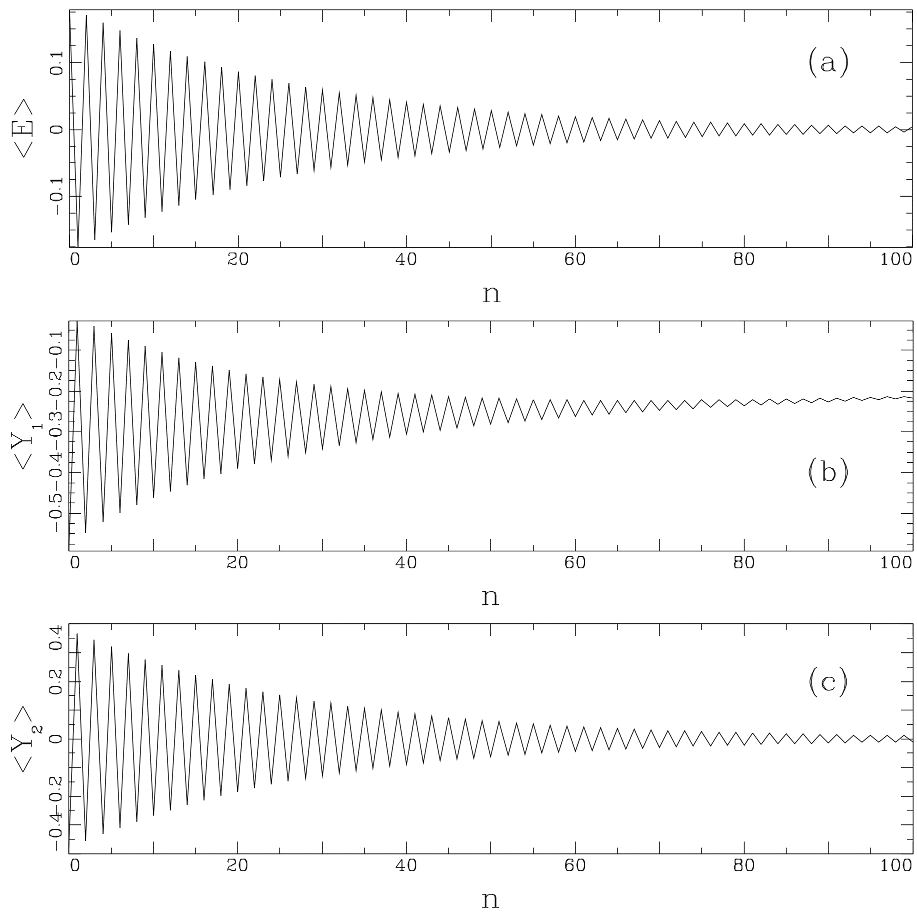
k-1
k
k+1
.....
4k-1
4k


(a)
$\langle E \rangle$
0.1
0
−0.1
0
20
40
60
80
100
n
(b)
$\langle Y_1 \rangle$
−0.1
−0.2
−0.3
−0.4
−0.5
0
20
40
60
80
100
n
(c)
$\langle Y_2 \rangle$
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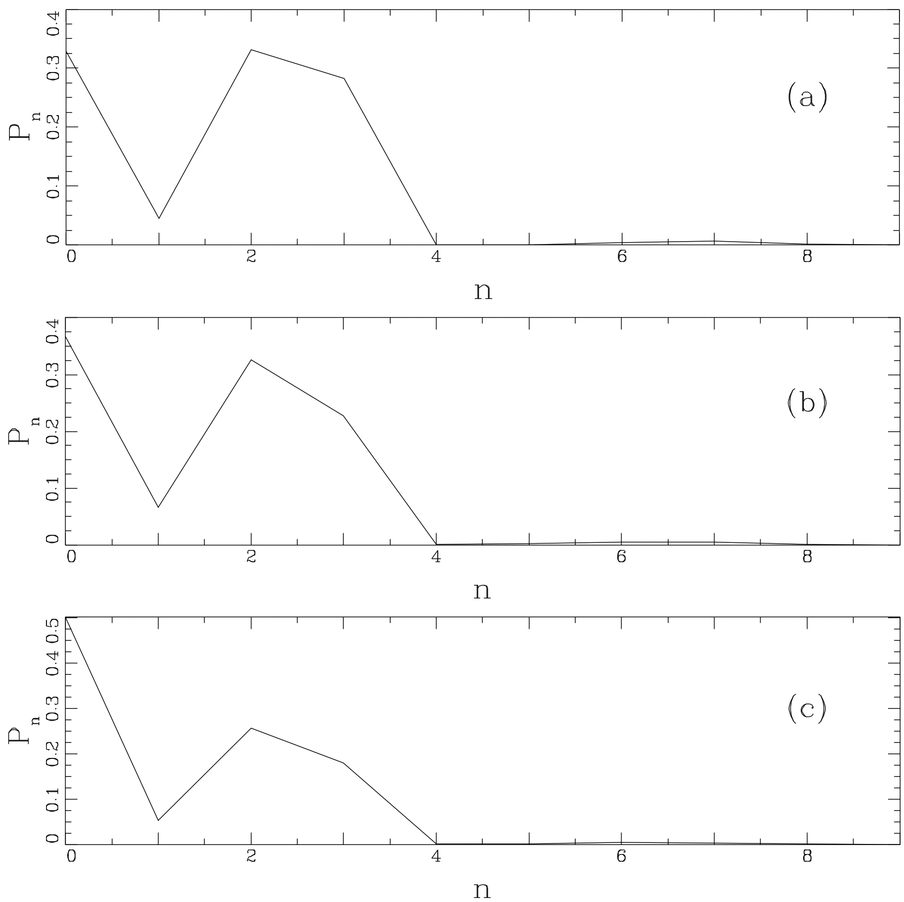
0.2
0
−0.2
−0.4
0
20
40
60
80
100
n



(a)
(b)
(c)
P
n
n
0
0.1
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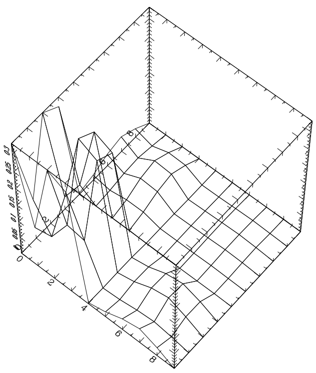
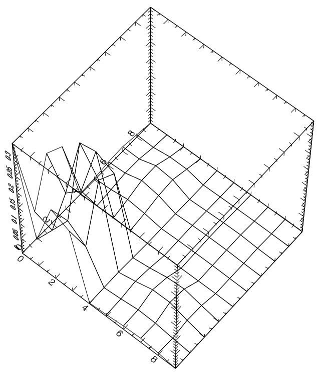
2
4
6
8



(a)



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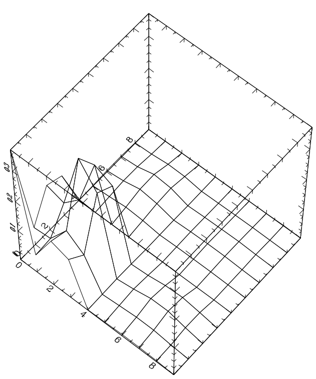
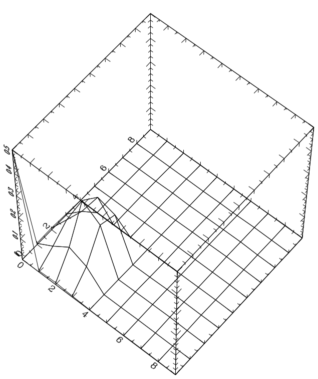


(c)



(d)